Solution of master equation for five-level ^{40}K atom

Jason McKeever

June 12, 2009

Updated June 12 2009, correction of wavelength λ_5

I have calculated the absorption cross section for driving of ^{40}K at 405 nm, on the 4S to 5P transition. Amir's report almost includes this information, but it isn't actually quantified in terms of the cross section. The calculation was inspired by the weak absorption (2-3 percent, even for multipass) seen experimentally in Dave's vapour cell. It was done using the Quantum Optics Toolbox for Matlab. The cross section is inferred from the steady state density matrix for the five level atom, so we also have access to the steady-state populations in the other excited levels. Using these results we can easily compute the fluorescence rates on the other transitions, for example.

All of the theory, decay rates and frequencies are taken from Amir's report (the equation to be solved is Eqn. 4). Exception: λ_5 is incorrect in Amir's report. I've changed it to be $1/\lambda_5 \equiv 1/\lambda_2 - 1/\lambda_3 - 1/\lambda_1$. This gives $\lambda_5 = 1177.4$ nm. Once the steady state density matrix for the atom is found, we have the equilibrium populations in all the levels. For each of the excited levels, we can then write the total rate of radiated energy as

$$P_{sc} = \rho_{ee} \sum_j \gamma_j \hbar \omega_j \tag{1}$$

where the sum is over all lower levels with an allowed dipole transition. This is done for all four excited states, and added up to get the total rate of scattered power. More specifically, in Amir's notation (see his Fig 2), we have

$$P_{sc} = \hbar \left[\rho_{55}(\gamma_4\omega_4 + \gamma_3\omega_3 + \gamma_2\omega_2) + \rho_{44}\gamma_6\omega_6 + \rho_{33}\gamma_5\omega_5 + \rho_{22}\gamma_1\omega_1 \right] \tag{2}$$

(the state labelling for the ρ_{nn} is the same as in Fig. 1). This is of course equal to the total rate of absorbed power, and then the cross section is obtained via

$$\sigma = \frac{P_{sc}}{I_{inc}} \tag{3}$$

where I_{inc} is the incident intensity.

It is interesting to compare the answer to the simple absorption cross section for a two level atom for $I \ll I_{sat}$,

$$\sigma_{2L} = \frac{3\lambda^2}{2\pi} \tag{4}$$

where $\lambda = 404.5$ nm. We can also compare to the expression derived by Dave, which basically gives a reduction in the cross section by the branching ratio for direct decay from 5P to 4S.

$$\sigma_{Dave} = \frac{P_{Dave}}{I_{inc}} \tag{5}$$

where

$$P_{Dave} = \rho_{55}(\gamma_4 + \gamma_3 + \gamma_2)\hbar\omega_2 \tag{6}$$

The main results are as follows. At low intensity $I \ll I_{sat}$, we have

$$\sigma = 0.174\ \sigma_{2L} \tag{7}$$

and

$$\sigma_{Dave} = 0.174\ \sigma_{2L} \tag{8}$$

We get the same answer from both approaches (to less than a part in 1000, where the discrepancy could come from being a bit off in the wavelengths etc). There's probably a fundamental reason why Dave's simpler approach gives the right answer, but I haven't thought this through.

It is also interesting to compare this cross section to that of the 4S − 4P transition at 767 nm:

$$\sigma^{(405)} = \frac{1}{21.6}\ \sigma^{(767)} \tag{9}$$

This suppression factor comes from product of the ratio of wavelengths squared with the $\approx 1/6$ coming from the multi-level cascade. This means that for the same vapour cell, assuming low intensity in both cases, we should have about 22 times less absorption in the blue system vs. the conventional IR one.

Another interesting point brought up by Dave: when doing Doppler-broadened spectroscopy, the situation is even worse (in terms of absorption at 405 nm vs 767 nm). The reason is basically that you get an extra factor of

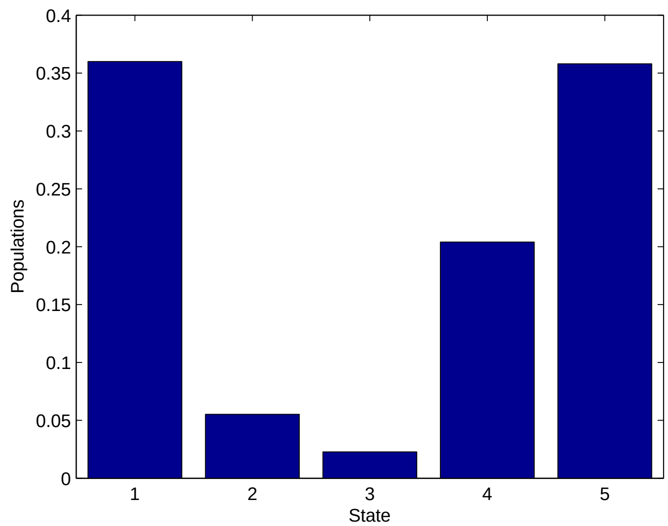

Figure 1: Population distribution for strong driving at 405 nm ($\Omega = 10^8\ \mathrm{s}^{-1}$). The state labelling is as follows: 1=4S, 2=4P, 3=3D, 4=5S and 5=5P.

Γ/Δ_D, where $\Delta_D \approx \mathrm{k}\overline{\mathrm{v}}$, and $\overline{\mathrm{v}}$ is the rms velocity of potassium in the vapour cell. Since at 405 nm Γ is smaller and k is larger, our suppression factor becomes more like 220 (source: Dave McKay, private communication).

For interest's sake, I have also included a bar graph of the internal state population distribution for strong driving (Fig. 1).